Extending Line-Line Relations

Shreyans Jain
School of Computing and Information Science
University of Maine
shreyans.jain@maine.edu

ABSTRACT

To capture the similarities between topological relations, conceptual neighborhood graphs have been used. A conceptual neighborhood graph for the thirty-three topological relations between two undirected lines has already been developed. However, it makes no further refinement on whether the interiors of the two lines touch, coincide, or cross each other. This paper extends the conceptual neighborhood graph of topological relations between two lines to distinguish between these different configurations.

1. INTRODUCTION

Spatial objects such as rivers, political boundaries and road networks can be modeled as lines in Geographic Information Systems (GISs) and spatial databases. "A line is a sequence of 1…. n connected 1-cells such that they neither cross themselves nor form a cycle" (Egenhofer, 1993). The nodes at which exactly one, 1-cell ends can be referred to as the *boundary* of the line. *Interior* nodes are the nodes which are endpoints of more than one 1-cell. The union of the *interior* and the *boundary* is called the *closure* for the line. The difference between the embedding space and the *closure* of the line is called the *exterior* of the line. A simple line is a line with two disconnected boundaries, whereas a complex line is a line with more than two disconnected boundaries. In this research, we will consider only the simple lines.

The simple lines can have a number of binary relationships among them. The topological relationships between two simple lines have been studied extensively. The 9-intersection model (Egenhofer and Herring, 1991) formally captures the topological relations between two spatial objects through the intersection of the objects' interiors, boundaries, and exteriors. Thus, the binary topological relation between any two spatial objects A and B in IR^2 is based on the comparison of A's interior ($A°$), boundary (∂A), and exterior (A^-) with B's interior ($B°$), boundary (∂B), and exterior (B^-).

$$R(A,B) = \begin{pmatrix} A° \cap B° & A° \cap \partial B & A° \cap B^- \\ \partial A \cap B° & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B° & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

Each intersection in the above matrix is categorized by a value, either: empty ($\emptyset$) or non-empty ($\neg\emptyset$).

For the 9-intersection model, the content of the 9 intersection matrix was recognized as the most general topological invariant (Egenhofer and Franzosa, 1991). Using this criterion it was revealed that there are only 33 possible binary spatial relations between two lines in IR^2 (Egenhofer, 1993). This approach of content invariance provides a solid framework which can be used to classify generic line-line relations. However, there can be a number of other topological invariants, which can be used to show more detailed differences in line-line relations. One such topological invariant can be the type of intersection, which can be touching, coinciding or crossing. Figure 1 illustrates three such scenarios for the same topological relationship (LL7) between two lines.

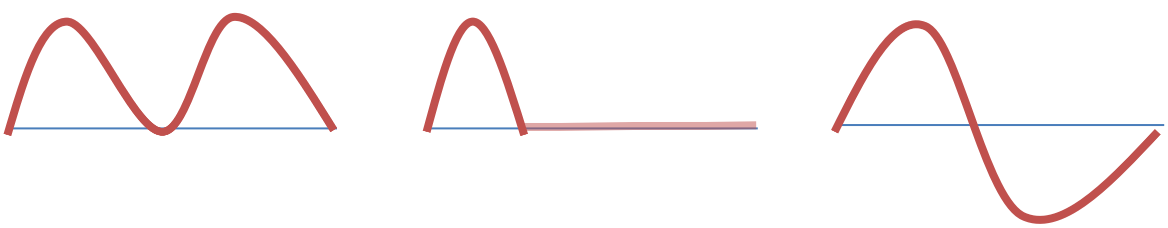
All the three configurations would have the same 9-intersection matrix described by:

$$R(A,B) = \begin{pmatrix} \neg\emptyset & \neg\emptyset & \neg\emptyset \\ \emptyset & \emptyset & \neg\emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{pmatrix}$$

(a) (b) (c)

Figure 1. Spatial relationships between two lines (a) Ravi river *touches* India-Pakistan border (b) Ravi river *coincides* with the border of the Indian states Jammu and Kashmir and Himachal Pradesh (c) Ravi river *crosses* India-Pakistan border.

To capture the similarities between topological relations, conceptual neighborhood graphs have been used. The nodes in a conceptual neighborhood graph represent spatial relations while the edges connect the most similar nodes. The first conceptual neighborhood graphs that have been studied were for the binary relations between intervals in R^1 (Freska, 1992) and binary topological relations between regions in R^2 (Egenhofer and Al-Taha, 1992). The conceptual neighborhood graph for two lines has also been studied for the 33 line-line relations (Reis, Egenhofer and Matos, 2008). While this conceptual neighborhood graph enables us to understand the similarities and dissimilarities between different topological relations, it is unable to capture the three distinctly different cases (touch, coincide, and cross) for the same topological relation (e.g. the different configurations of the same topological line-line relation LL23 in figure 2). We thus consider the ramifications of considering these relations as distinct, rather than just a single class.



(a) (b) (c)

Figure 2. Two lines (a) touching, (b) coinciding, and (c) crossing each other.

This paper extends the conceptual neighborhood graph of binary topological relations between lines, so as to also include the intermediate relations and alternative paths between two lines for different configurations. The remainder of this paper is organized as follows: Section 2 reviews the related work on conceptual neighborhood graph for line-line relations. Section 3 develops the change process involved when the two lines alter from the states of touch, coincide and cross and also derives the similarities between the relations. Section 4 offers conclusions and Section 5 discusses future work.

2. RELATED WORK

The 9-intersection model identifies 33 topological relations between two undirected line segments. The conceptual neighborhood graph for simple line-line relations (Reis, Egenhofer and Matos, 2008) is shown in Figure 3. This graph consists of two connected layers. Each of these layers has 13 nodes along with a pointed node. There is also a reduced third layer with another pointed node consisting of a total of 5 nodes. Layers 2 and 3 have non-empty interior-interior intersections. While this graph captures the similarities between two binary topological relations, it does not capture the different configurations (namely touch, coincide and cross) that are possible when the two lines have non-empty interior-interior intersection. This paper extends the conceptual neighborhood graph for binary line-line relations by also incorporating the spatial relation of two lines when they touch, coincide or cross each other as distinct cases.

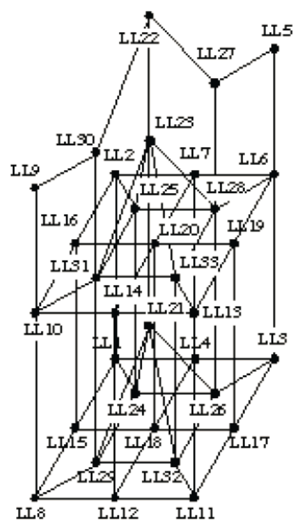
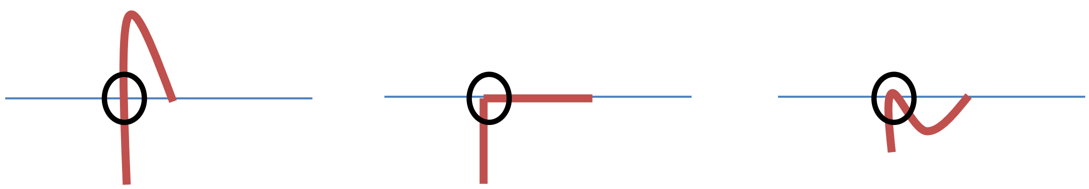


Figure 3. The conceptual neighborhood graph of thirty three topological relations between two lines (Reis, Egenhofer and Matos, 2008).

3. CONCEPTUAL NEIGHBORHOOD GRAPH FOR TOPOLOGICAL RELATIONS BETWEEN TWO UNDIRECTED LINES CONSIDERING DIFFERENT CONFIGURATIONS

In this section, we will consider the conceptual neighborhoods of line-line spatial relations with additional refinements. To capture the conceptual neighborhood graph for two lines when they touch, coincide or cross each other, we need to consider only those spatial relations where the interior-interior intersection between the lines is non-empty. As we know, layers 2 and 3 in the conceptual neighborhood graph of two lines (Figure 3) have non-empty interior-interior intersections. We begin by considering the similarities for line-line spatial relations in layer 2. We will then see how these conceptual neighborhood graphs form linkages to layer 1 and layer 3.

We can distinguish between the three cases by considering the sequence of lines we encounter while traversing clockwise, at the intersection. The “cross” configuration would have the order as a cyclic permutation of the following - 1st object, 2nd object, 1st object, 2nd object. The “coincide” configuration would have the order, a cyclic permutation of - 1st object, (1st and 2nd object), 2nd object. Finally, the “touch” configuration would be a cyclic permutation of- 1st object, 2nd object, 2nd object, 1st object. Figure 4 gives an example for these different configurations for the line-line relation LL7.



(a) **(b)** **(c)**

Figure 4. The three possible configuration for binary spatial relation, LL7- (a) LL7 Cross- Red, Blue, Red, Blue , (b) LL7 Coincide- Blue, Red-Blue, Red and (c) LL7 Touch- Blue, Red, Red, Blue.

5.1 Conceptual Neighborhood Graph for the relations in extended layer 2

To investigate the conceptual neighborhood graph for extended line-line spatial relations, we begin by examining LL2, which lies on the back left corner of the layer under consideration. We start with the touch configuration represented as LL2T. We pull the curved line so that one of the boundaries now touches the interior of the straight line. If we now pull this line straight we reach the cross configuration, LL2X (Figure 5). However, we notice that there is no coincide configuration possible for LL2 as both lines start and end in the exterior. We, also notice that the intermediate case between LL2T and LL2X is LL7C, the coincide configuration for line-line relation 7.

In LL2T, if we pull one of the boundary points of the thicker line to merge with the thinner line, we get a new spatial relation, LL7. This relation follows the touch configuration of the parent relation and hence is labeled as LL7T. If we now rotate the thicker line to the thinner line we reach the coincide version of the LL7 spatial relation. Pulling the vertex up, in LL7C would lead us to the cross version of LL7 labeled as LL7X in figure 5 (i).

We can continue this process of rotating and pulling a line and can thus extend the back row of layer 2 in the conceptual neighborhood graph of line-line spatial relations to the conceptual neighborhood graph in figure 5 (i).

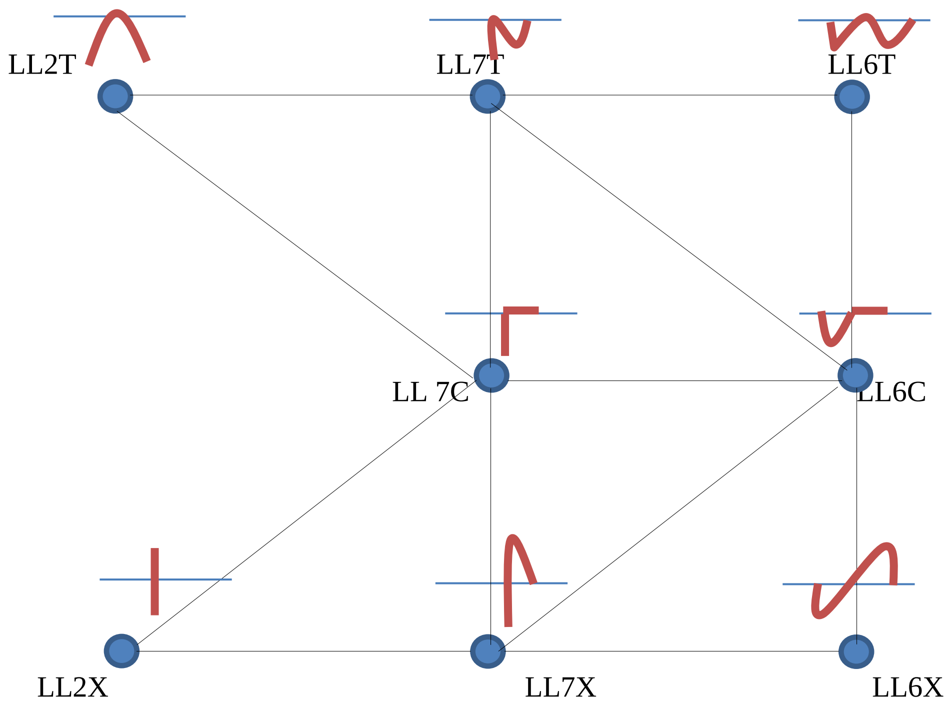


Figure 5 (i) Conceptual neighborhood graph for line-line spatial relations, LL2, LL7 and LL6, considering the refinement of whether the lines touch, coincide or cross each other.

We can thus extend the entire layer of the conceptual neighborhood graph of line-line relations with non-empty interior-interior intersections to include the additional refinement of whether the lines touch, coincide or cross each other. The following series of mappings, Figures 5 (ii) – (xiv) explore the other sets of connections within the second layer of the conceptual neighborhood graph.

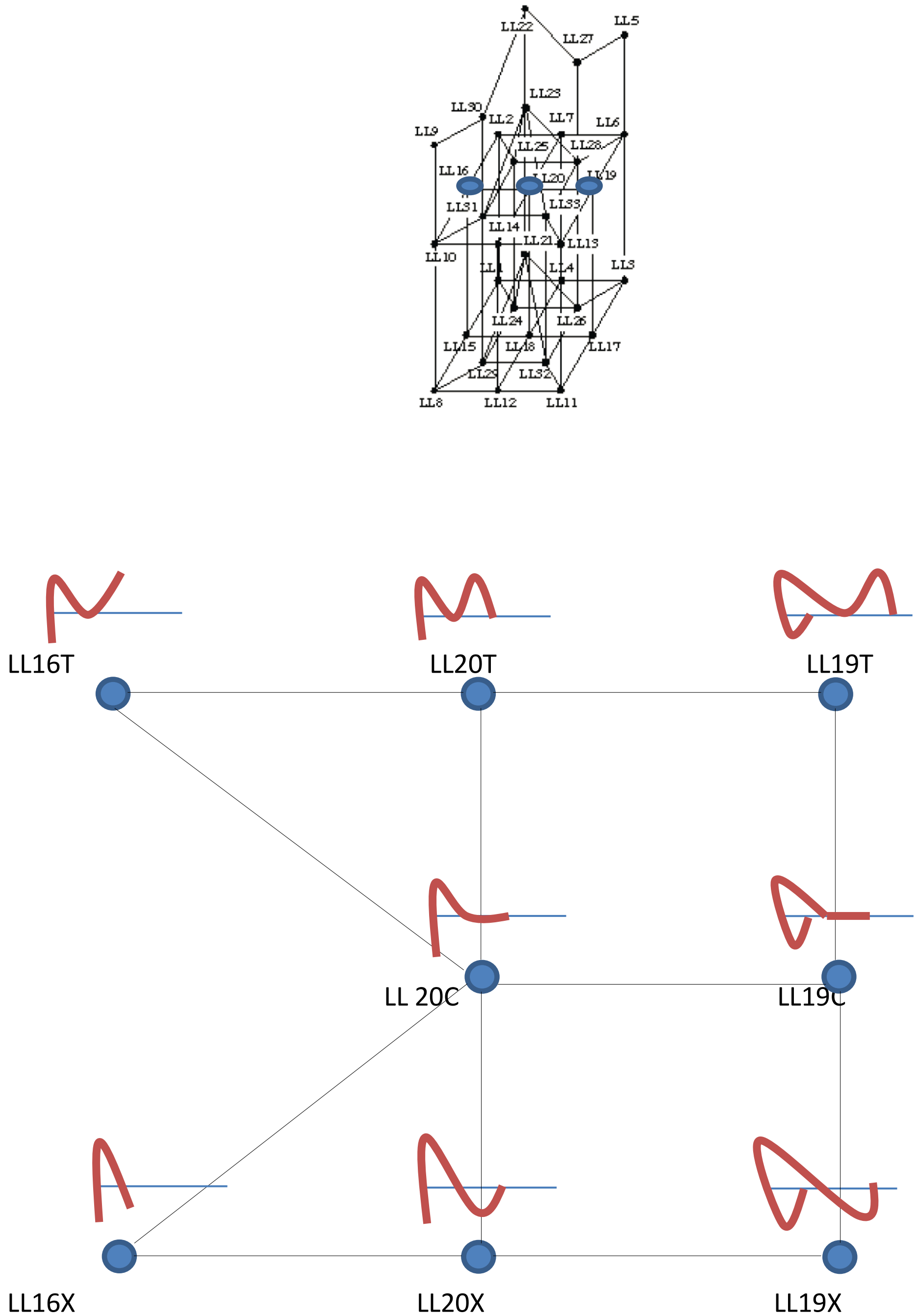

LL22
LL5
LL27
LL23
LL30
LL2
LL7
LL6
LL9
LL25
LL28
LL16
LL20
LL19
LL31
LL33
LL14
LL21
LL13
LL10
LL1
LL4
LL3
LL24
LL26
LL15
LL18
LL17
LL29
LL32
LL8
LL12
LL11
LL16T
LL20T
LL19T
LL 20C
LL19C
LL16X
LL20X
LL19X

Figure 5 (ii)

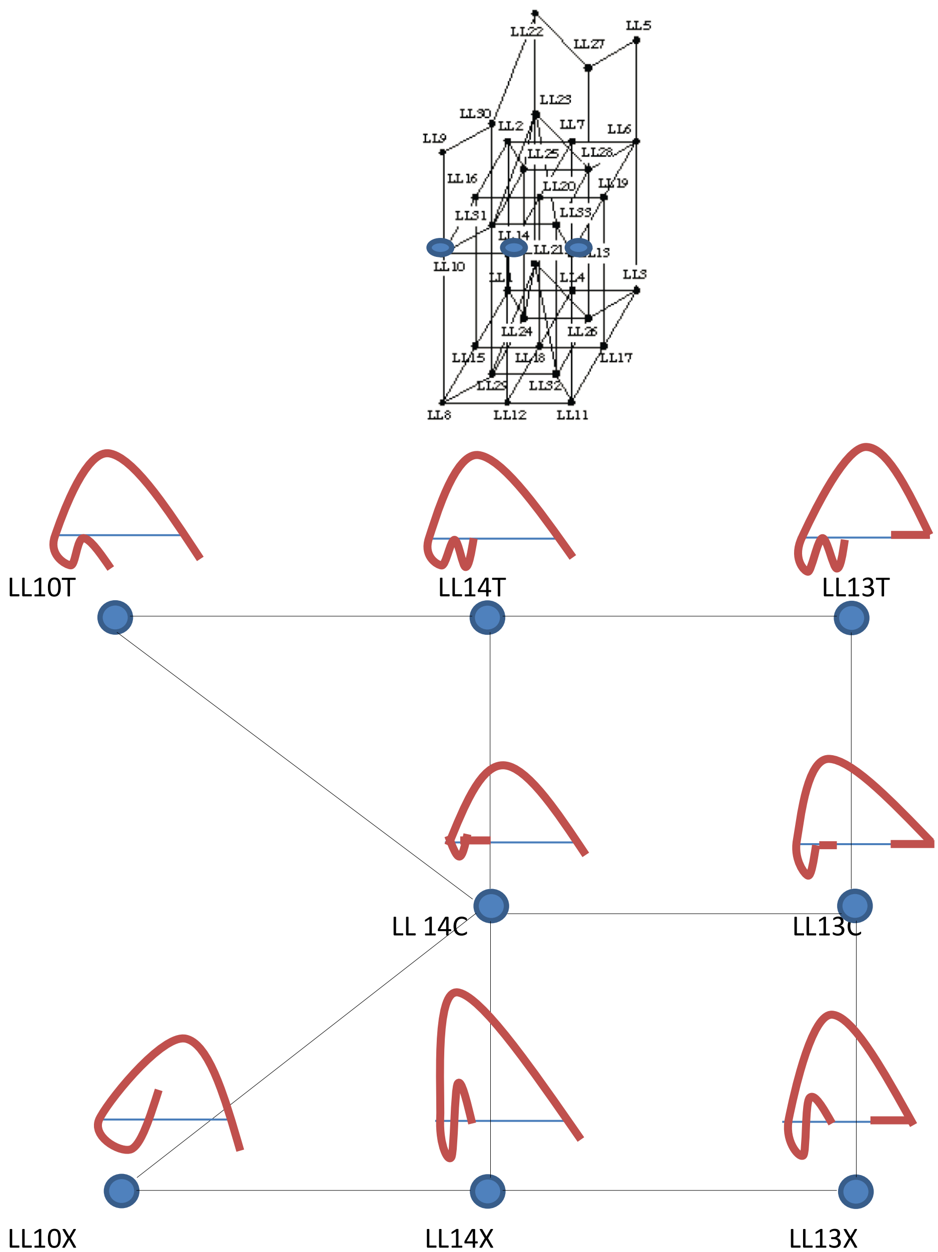

LL22
LL27
LL5
LL23
LL30
LL2
LL7
LL6
LL9
LL25
LL28
LL16
LL20
LL19
LL31
LL33
LL14
LL21
LL13
LL10
LL4
LL3
LL24
LL26
LL15
LL18
LL17
LL29
LL32
LL8
LL12
LL11
LL10T
LL14T
LL13T
LL 14C
LL13C
LL10X
LL14X
LL13X

Figure 5 (iii)

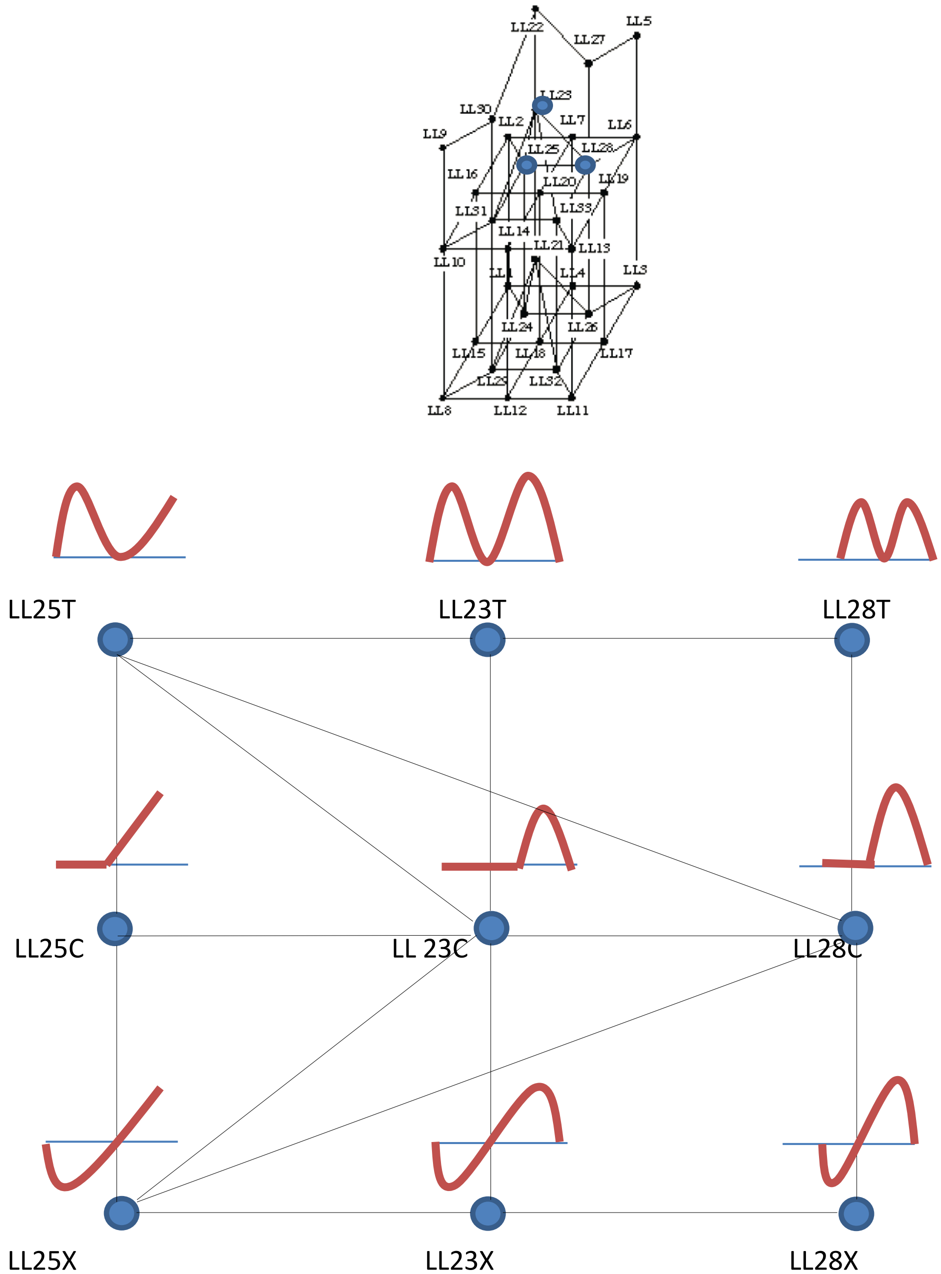


Figure 5 (iv)

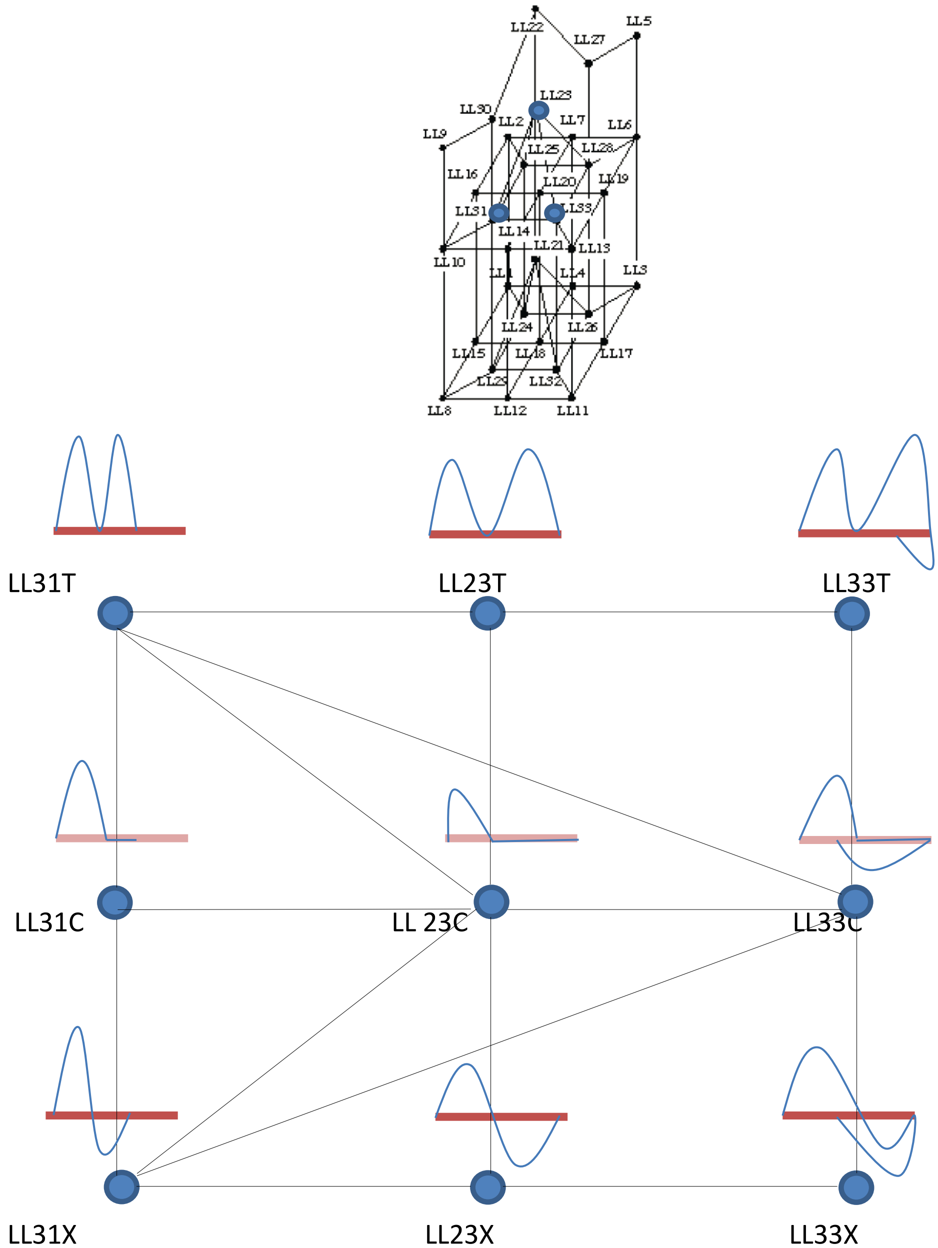

LL22
LL5
LL27
LL23
LL30
LL2
LL7
LL6
LL9
LL25
LL28
LL16
LL20
LL19
LL31
LL33
LL14
LL21
LL13
LL10
LL1
LL4
LL3
LL24
LL26
LL15
LL18
LL17
LL29
LL32
LL8
LL12
LL11
LL31T
LL23T
LL33T
LL31C
LL 23C
LL33C
LL31X
LL23X
LL33X

Figure 5 (v)

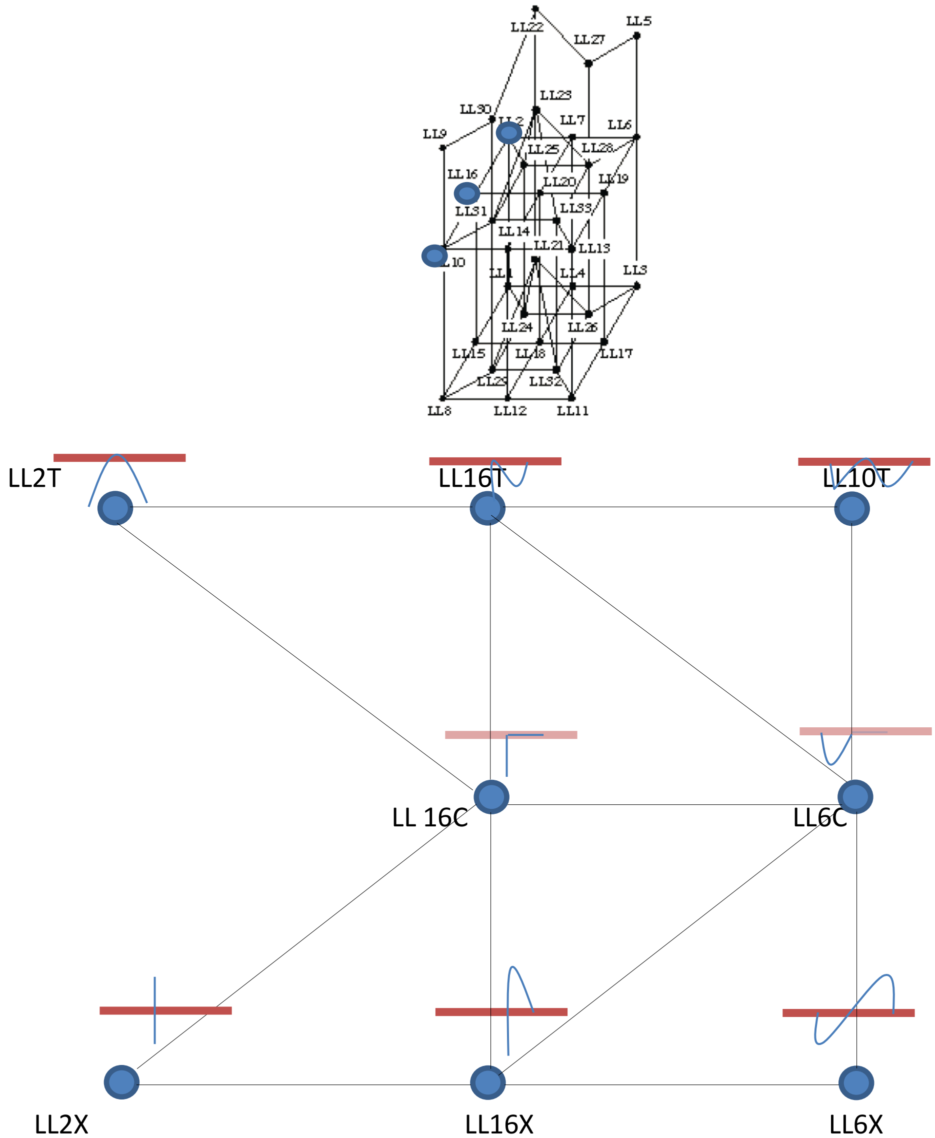


Figure 5 (vi)

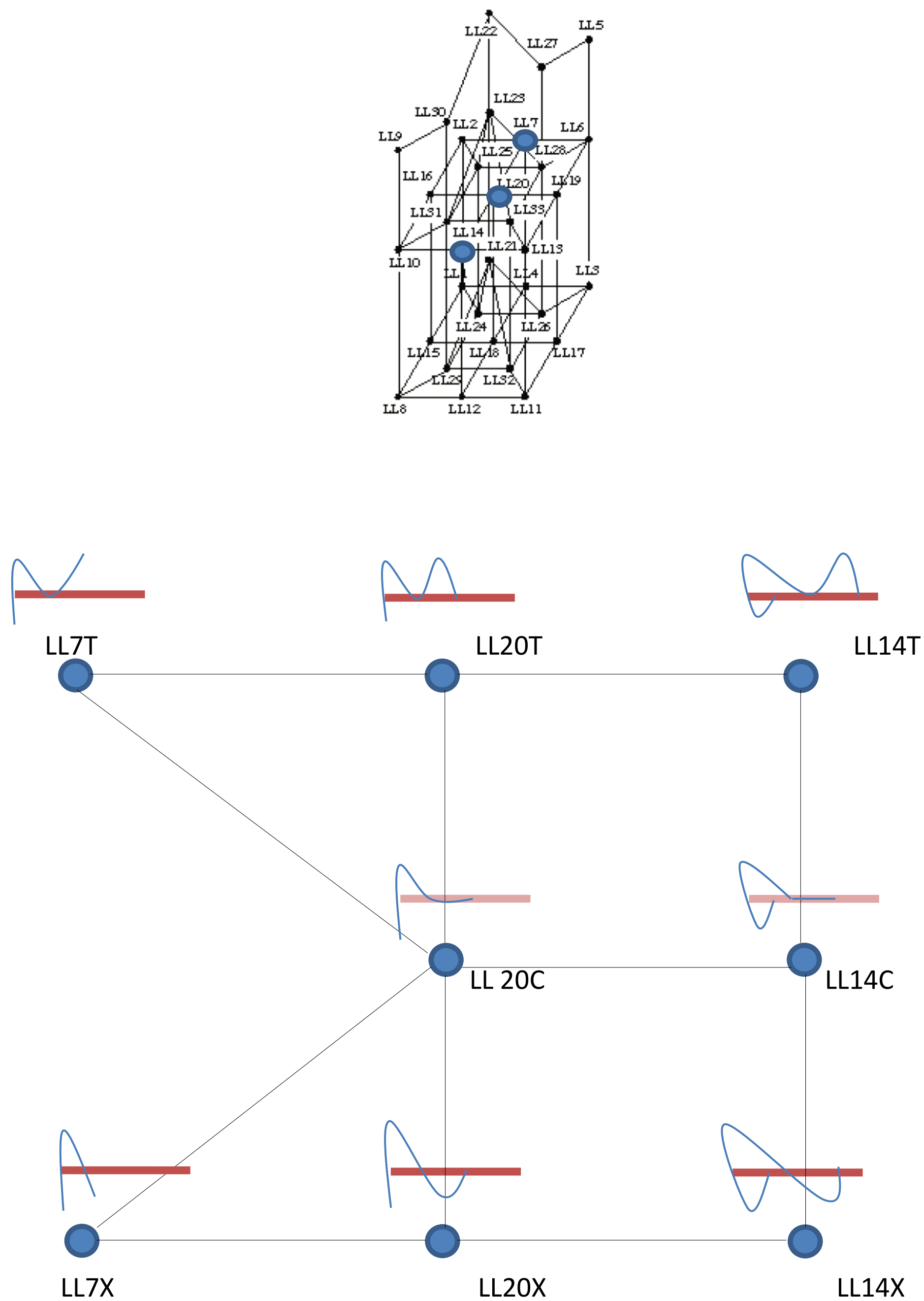

LL22
LL27
LL5
LL23
LL30
LL2
LL7
LL6
LL9
LL25
LL28
LL16
LL20
LL19
LL31
LL33
LL14
LL21
LL13
LL10
LL4
LL3
LL24
LL26
LL15
LL18
LL17
LL29
LL32
LL8
LL12
LL11
LL7T
LL20T
LL14T
LL 20C
LL14C
LL7X
LL20X
LL14X

Figure 5 (vii)

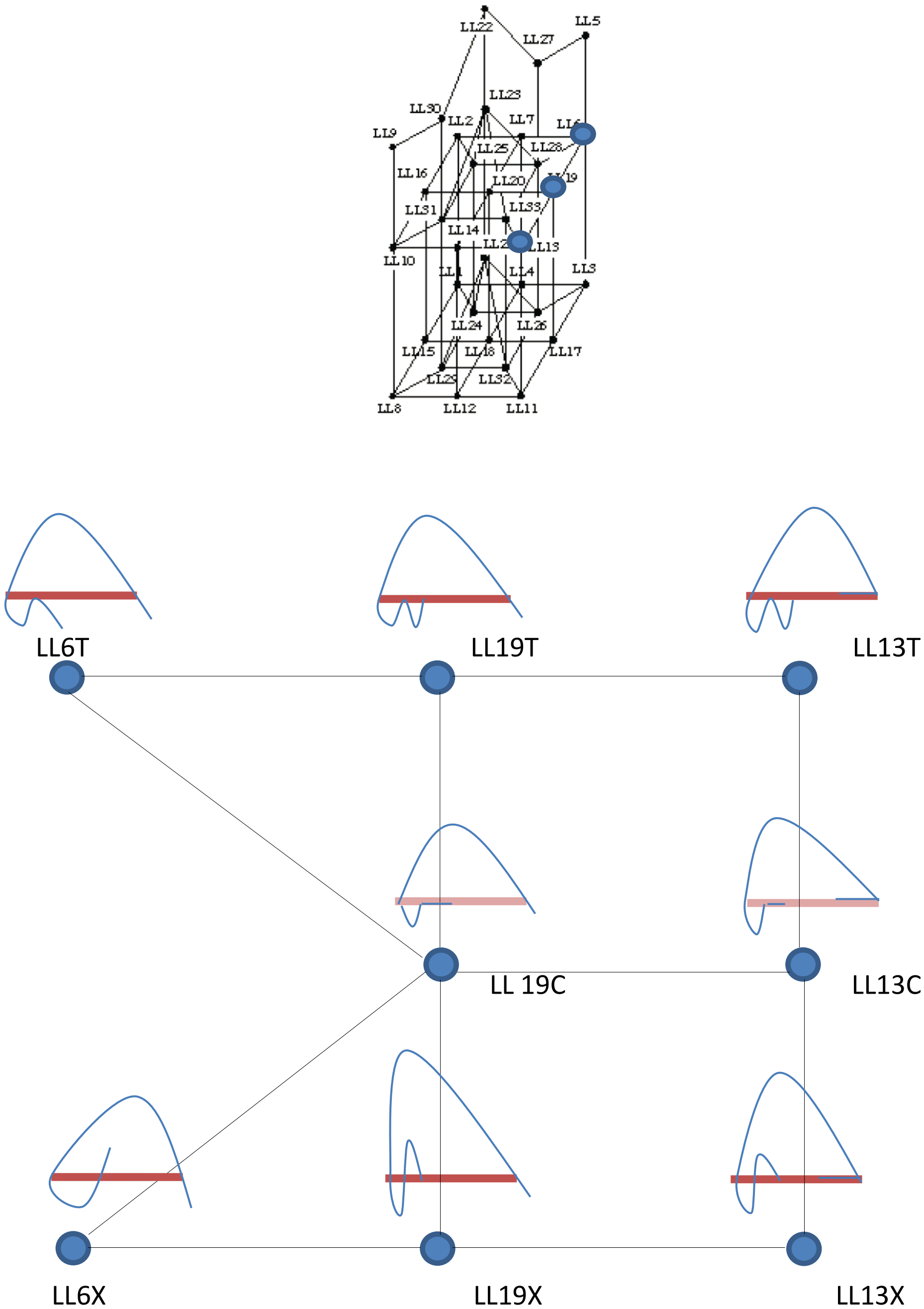


Figure 5 (viii)

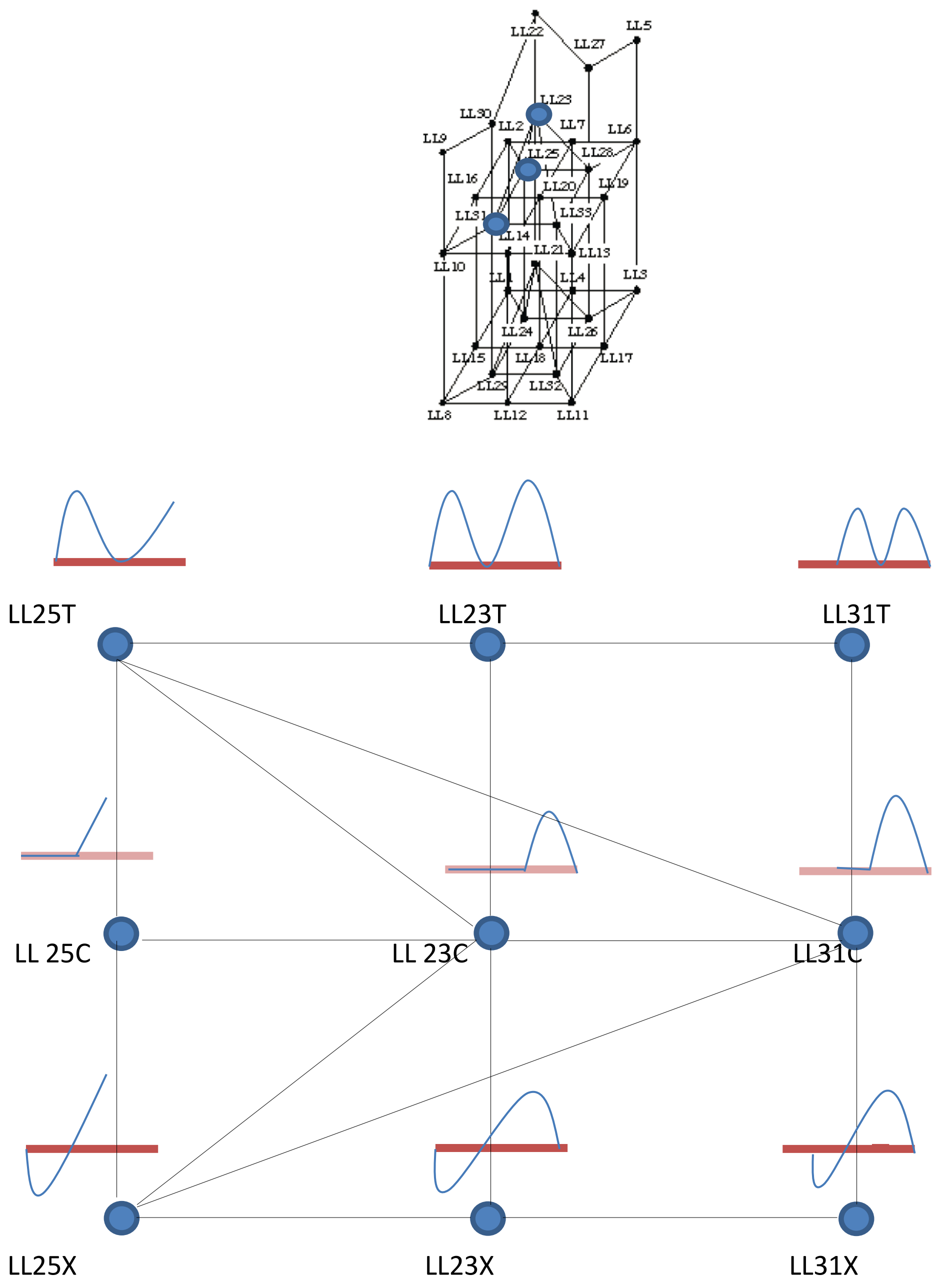


Figure 5 (ix)

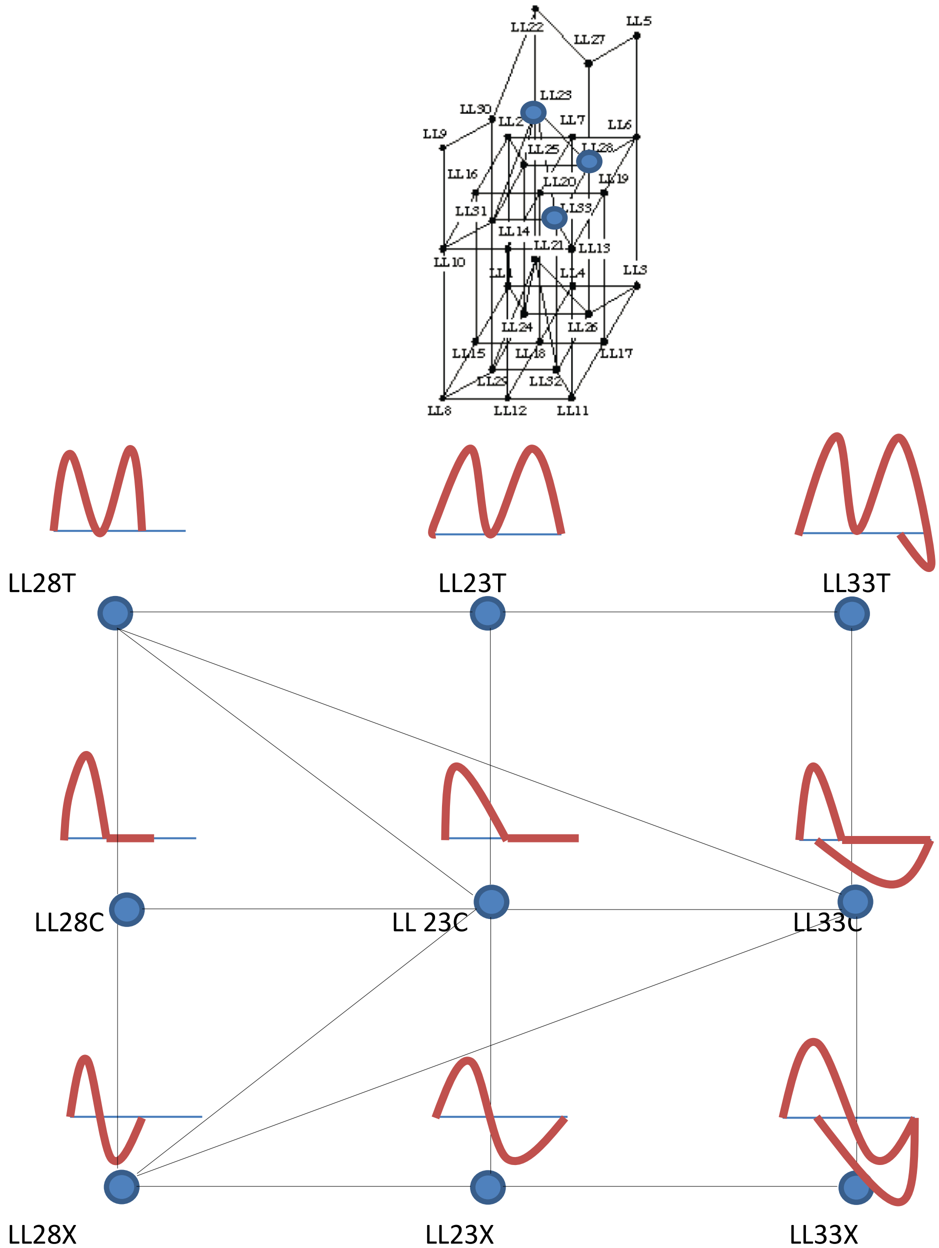


Figure 5 (x)

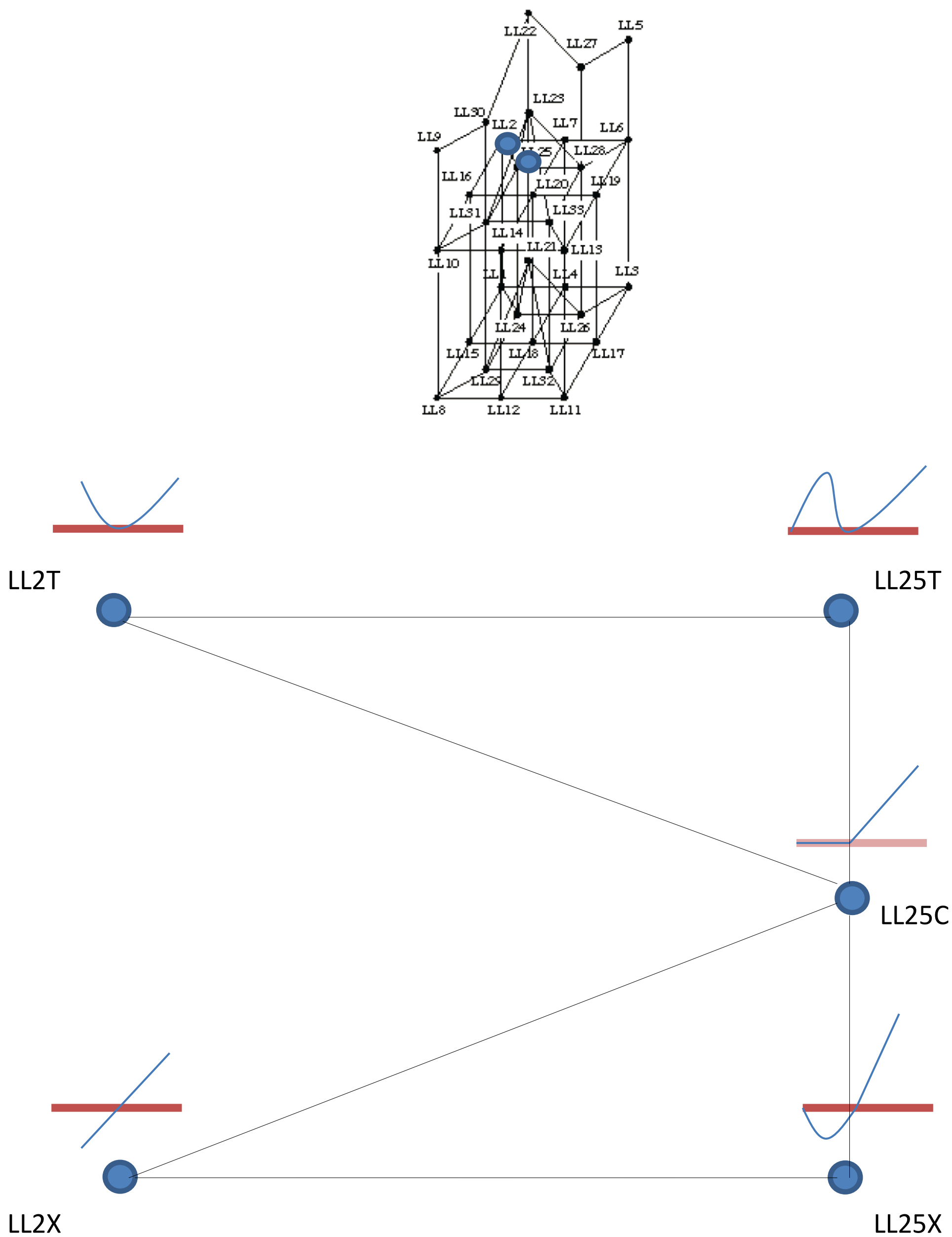
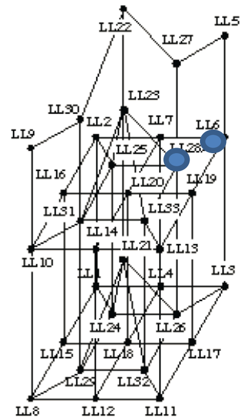


Figure 5 (xi)


LL22
LL27
LL5
LL23
LL30
LL2
LL7
LL6
LL9
LL25
LL28
LL16
LL20
LL19
LL31
LL33
LL14
LL21
LL13
LL10
LL1
LL4
LL3
LL24
LL26
LL15
LL18
LL17
LL29
LL32
LL8
LL12
LL11

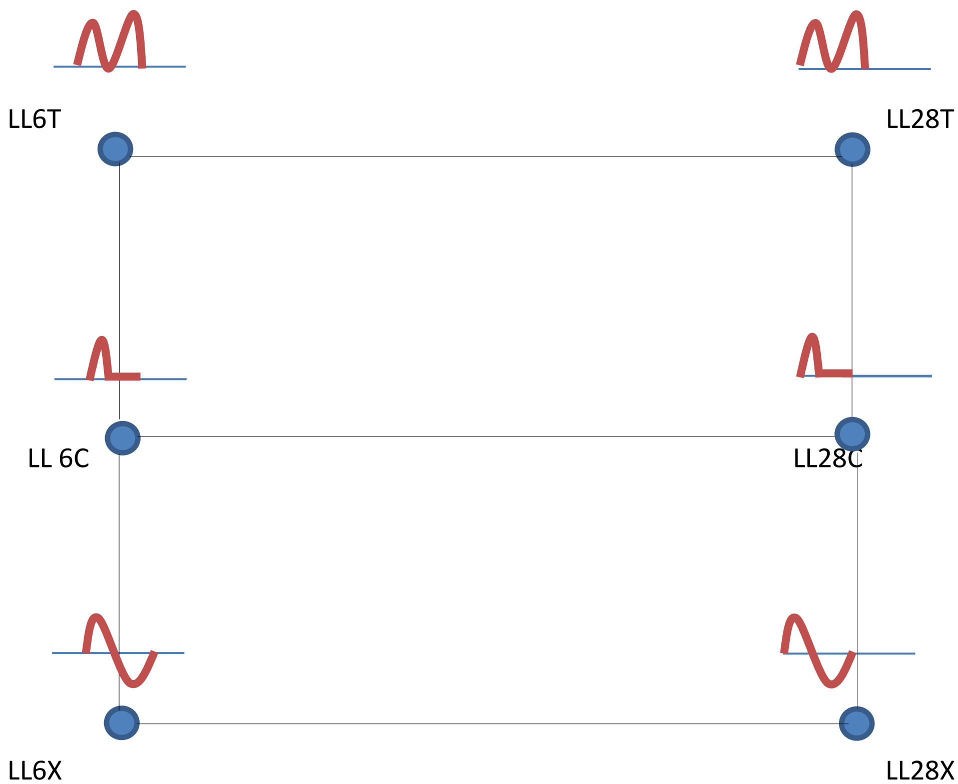

LL6T
LL28T
LL 6C
LL28C
LL6X
LL28X

Figure 5 (xii)

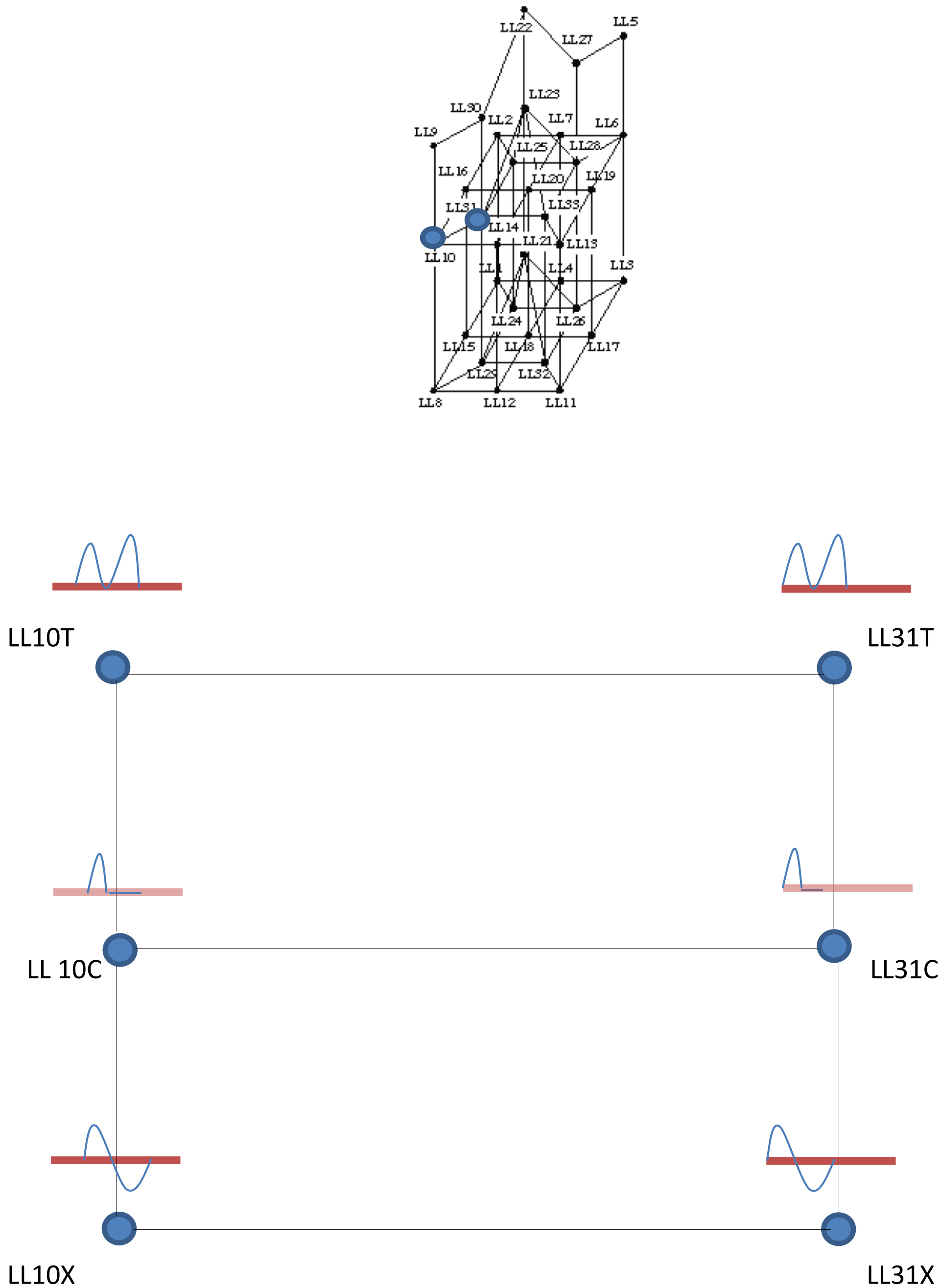


Figure 5 (xiii)

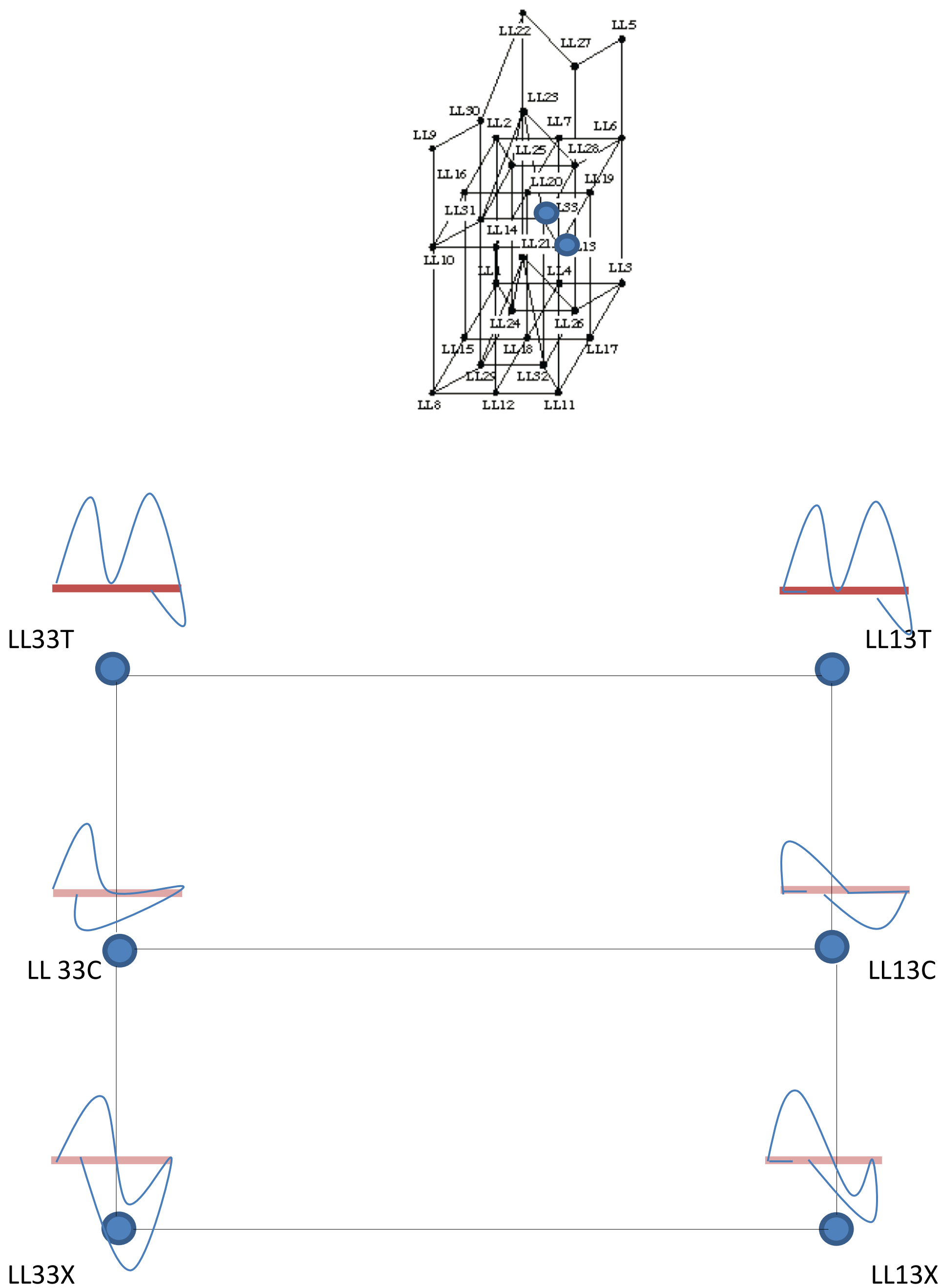


Figure 5 (xiv)

5.2 Conceptual Neighborhood Graph for the spatial relations in layer 3 and extended layer 2

The Layer 3 of the conceptual neighborhood graph for line-line relation has a set of five nodes. These five relations are equal (LL 22), starts and finishes (LL 27 and LL 30) and the true subset with its converse relation (LL5 and LL9). We begin our investigation, by examining the left most relation in the third layer (LL9). We notice that in Figure 6 (i) by pulling a point up in the LL9 configuration, we reach the coincide version of LL10.

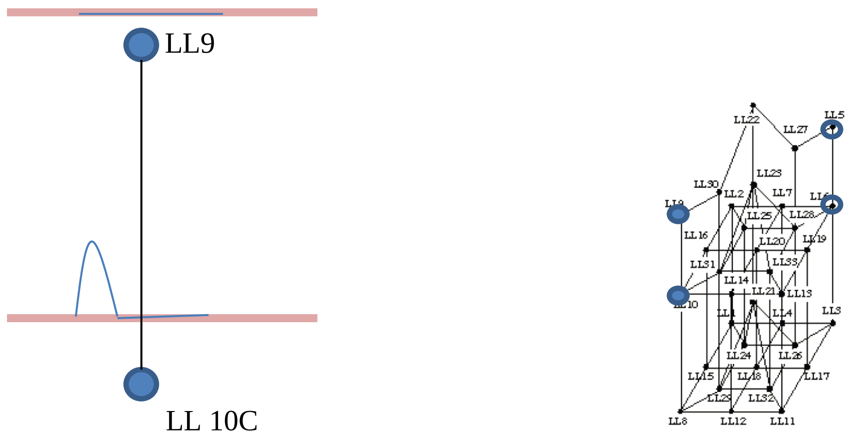


Figure 6 (i) Conceptual Neighborhood links between LL 9 and LL10C and their converse.

We find similar linkage between LL30 and LL31C as shown in Figure 6 (ii). Since LL27 and LL28 are converse relations to LL30 and LL31, we can conclude that LL27 and LL28C would have the same linkage. Also, LL5 and LL6C being converse of LL9 and LL10C, respectively would have identical link.

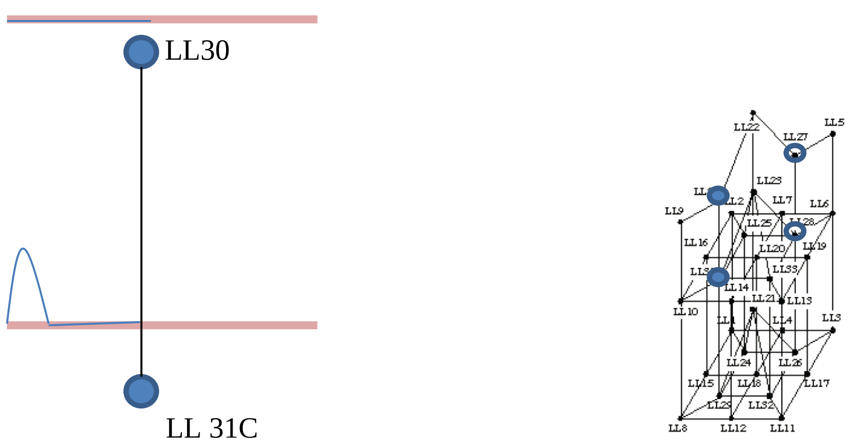


Figure 6 (ii) Conceptual Neighborhood link between LL 30 and LL31C and their converse.

In the same way LL22 is connected to LL23C as shown in Figure 6 (iii).

Figure 6 (iii) Conceptual Neighborhood link between LL 22 and LL23C and their converse.

5.3 Conceptual Neighborhood Graph for the relations in layer 1 and extended layer 2

Layer 1 of the conceptual neighborhood contains all the spatial relations which do not have an interior-interior intersection. These relations can be reached by pulling the point of interior-interior intersection from layer 2 to the exterior. Hence a relation in Layer 1 of the conceptual neighborhood graph of the line-line relation, can be reached from the touch configuration of the line-line relation, just above it. Figure 7, shows the graphical representation of one such conceptual neighborhood between the relations-LL8 in layer 1, with the touch configuration of LL10.

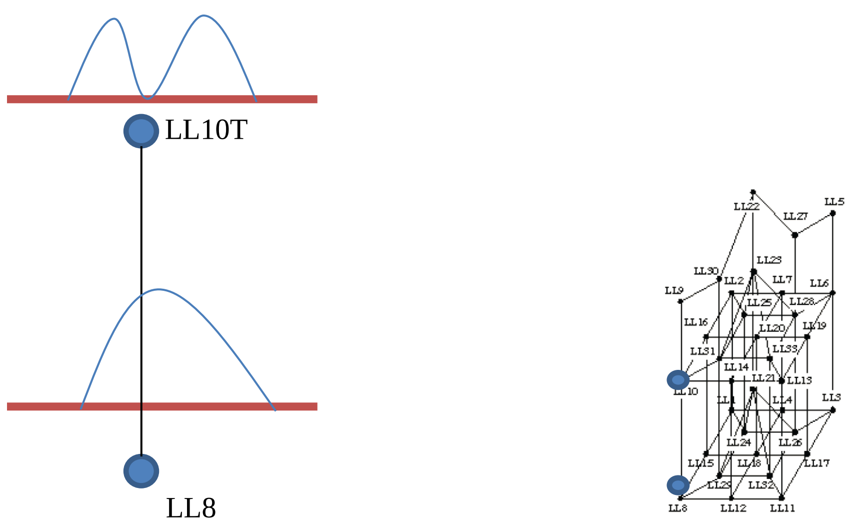


Figure 7 Conceptual Neighborhood link between LL 8 and LL10T.

4. CONCLUSIONS

The conceptual neighborhood graph for topological relations between two lines was extended to incorporate cases when the lines touch, coincide or cross each other. All line-line relations except LL2, have a coincide case, which acts as a bridge between the touch and cross configurations. Overall, the extended neighborhood graph follows the same neighborhood patterns as the original graph, which verifies the correctness of the original conceptual neighborhood graph for line-line spatial relations.

We found 5 distinct patterns of sub-graphs, while extending the original conceptual neighborhood graph for binary topological relations between lines. The distinct patterns arise due to the lack of a coincide case for LL2 spatial relation.

The extended conceptual neighborhood graph retains the overall neighborhood and structural characteristics of the original graph for line-line relations. To summarize, we enlist the salient features of the new graph-

- All line-line spatial relations in the second layer have three possible configurations - touch, coincide and cross, which are described by the same matrices by the original 9 intersection model, with an exception of LL2 which has only two possible configurations- touch and cross.
- The touch, coincide and cross configurations of a spatial relation are connected to the touch, coincide and cross configuration of the neighbor respectively.
- We find that the touch configuration and the cross configuration for a line-line spatial relation are not conceptual neighbors. However, they are connected by the coincide configuration which is a neighbor of both the touch as well as cross configuration.
- The line-line relation 2 (LL2) does not have a coincide case. This is because of the fact that the only intersection of each line's boundary is with the exterior of the other line.
- We find that the touch configuration for line-line relations in layer 2 has a conceptual neighborhood link to layer 1. Similarly, the coincide configurations for the binary topological relations in layer 2 have links to layer 3.

Being able to distinguish between cases, when two lines touch, coincide or cross each other would lead to improvements in geographic information systems. These refinements would allow us to store features like roads, political boundaries, rivers etc. and the relationships between them with greater details.

5. FUTURE WORK

We extended the conceptual neighborhood graph for the 33 topological relations between two undirected lines. The same methodology can be followed to extend the conceptual neighborhood graph for the 68 topological relations between two directed lines in R^2.

6. ACKNOWLEDGMENTS

The author would like to acknowledge Dr. Max Egenhofer and Matt Dube for their support in this work.

7. REFERENCES

Egenhofer M. , Franzosa R. (1991). “Point-Set Topological Spatial Relations”, International Journal of Geographic Information Systems, 5(2), 161–174.

Egenhofer, M., and Herring, J. (1991). “Categorizing Binary Topological Relationships Between Regions, Lines, and Points in Geographic Databases”, Department of Surveying Engineering, University of Maine, Orono, ME.

Egenhofer, M., Al-Taha, K.(1992). “Reasoning about Gradual Changes of Topological Relationships”, In: Frank, A., Campari, I., Formentini, U. (eds.): Theories and Methods of Spatio-Temporal Reasoning in Geographic Space, Lecture Notes in Computer Science, Springer, Berlin/Heidelberg, Germany , 639, 196-219.

Egenhofer, M. (1993). “Definition of Line-Line Relations for Geographic Databases”, Data Engineering, 16(6),
40-45.

Freska, C. (1992). “Temporal Reasoning based on Semi-Intervals”, Artificial Intelligence, 54, 199-227.

Reis, R., Egenhofer, M., Matos, J. (2008) “Conceptual Neighborhoods of Topological Relations between Lines”, In: Ruas, A., Gold, C. (eds.): 13th International Symposium on Spatial Data Handling, Headway in Spatial Data Handling, Lecture Notes in Geoinformation and Cartography, 557-574